Regret Bounds for Lifelong Learning

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1. Transfer learning, multitask learning, lifelong learning...
2. A strategy for lifelong learning, with regret analysis
3. Open questions

Generic learning task

A generic learning task

Given pairs object-label

$$(X_1, Y_1), \ldots, (X_n, Y_n)$$

learn to predict labels from objects.

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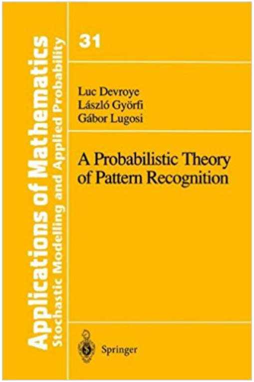
- ...

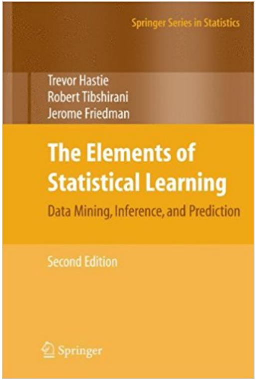
Batch learning

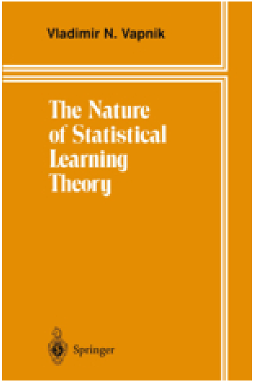
- data often assumed i.i.d from P,
- build $\hat{f}$ based on the whole dataset,
- minimize $R(\hat{f})$ where

$$R(f) = \mathbb{E}_{(X,Y)\sim P}[\ell(Y, f(X))].$$



Batch learning : more books

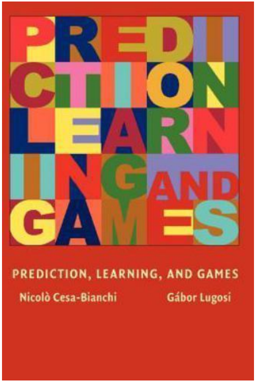




Online learning

- no probabilistic assumption,
- data revealed sequentially, at time t build $\hat{f}_t$ based on data seen so far
- minimize

$$\sum_{t=1}^{T} \ell(Y_t, \hat{f}_t(X_t))$$



Online learning : a good starting point

Foundations and Trends® in
Machine Learning
Vol. 4, No. 2 (2011) 107–194
© 2012 S. Shalev-Shwartz
DOI: 10.1561/2200000018

now
the essence of knowledge

Online Learning and Online Convex Optimization

By Shai Shalev-Shwartz

Contents

A few facts - motivation for transfer learning

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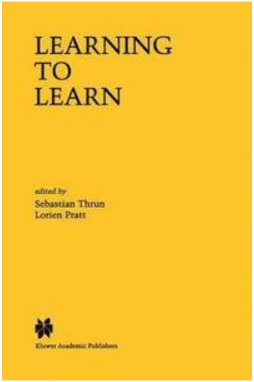
A few facts - motivation for transfer learning

- when we solve different tasks, it seems we start from scratch at each task
- still, our knowledge on "solving tasks" improves at each time
- for similar task, it seems indeed reasonnable to transfer information from one task to another.

Tentative definition - from Thrun and Pratt

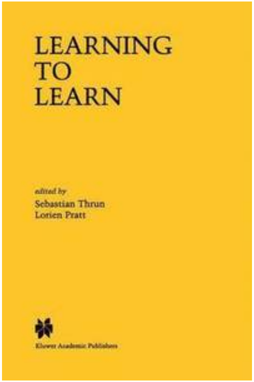


Given

- a task,
- a training experience, and
- a performance measure,

a program is said to learn if its performance at the task improves with experience.

Tentative definition - from Thrun and Pratt



Given

- a **family of** tasks,
- training experience for each of these tasks, and
- a family of performance measures,

an algorithm is said to **learn to learn** if its performance at each task improve with experience **and with the number of tasks.**

Multitask learning

Multitask learning

Given M tasks t, with M risks $R_t(\cdot)$ and M datasets

$$\mathcal{S}_t := \Big((X_{t,1}, Y_{t,1}), \dots, (X_{t,n_M}, Y_{t,n_M}) \Big)$$

propose M predictors

$$\hat{f}_t(\cdot) = \hat{f}_t(\mathcal{S}_1, \dots, \mathcal{S}_M; \cdot)$$

that aims at minimizing (for example)

$$R_1(\hat{f}_1) + \dots + R_M(\hat{f}_M).$$

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Nice, but what if yet another new task appears ?

Learning-to-learn

Learning-to-learn (LTL)

Given M tasks t with risk $R_t(\cdot)$, and M datasets

$$\mathcal{S}_t := \Big((X_{t,1}, Y_{t,1}), \dots, (X_{t,n_M}, Y_{t,n_M})\Big)$$

learn information $\mathcal{I} = \mathcal{I}(\mathcal{S}_1, \dots, \mathcal{S}_M)$ such that, when a **new** task with risk $R(\cdot)$ and a new dataset

$$\mathcal{S} := \Big((X_1, Y_1), \dots, (X_n, Y_n)\Big)$$

arrives, I can build a predictor

$$\hat{f}_t(\cdot) = \hat{f}_t(\mathcal{S}, \mathcal{I}; \cdot) \text{ such that } R(\hat{f}) \text{ is small.}$$

Probabilistic setting for LTL

Possible probabilistic setting :

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- $P_1, \dots, P_M$ i.i.d from $\mathcal{P}$,

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- $R_t(f) = \mathbb{E}_{(X,Y)\sim P_t}[\ell(Y, f(X))]$,
- quantitative criterion to minimize w.r.t $\mathcal{I}$

$$\mathcal{R}_{\mathrm{LTL}}(\mathcal{I}) = \mathbb{E}_{P\sim\mathcal{P}} \left\{ \min_{f\in\mathcal{C}} \mathbb{E}_{(X,Y)\sim P} \left[\ell(Y, f(\mathcal{I}, X))\right] \right\}.$$

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Note the strong Bayesian flavor...

Example of LTL : dictionary learning

Example taken from :

PMLR Proceedings of Machine Learning Research

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Sparse coding for multitask and transfer learning

[edit]

Andreas Maurer, Massi Pontil, Bernardino Romera-Paredes ; Proceedings of the 30th International Conference on Machine Learning, PMLR 28(2):343-351, 2013.

Abstract

We investigate the use of sparse coding and dictionary learning in the context of multitask and transfer learning. The central assumption of our learning method is that the tasks parameters are well approximated by sparse linear combinations of the atoms of a dictionary on a high or infinite dimensional space. This assumption, together with the large quantity of available data in the multitask and transfer learning settings, allows a principled choice of the dictionary. We provide bounds on the generalization error of this approach, for both settings. Numerical experiments on one synthetic and two real datasets show the advantage of our method over single task learning, a previous method based on orthogonal and dense representation of the tasks and a related method learning task grouping.

Related Material

- Download PDF
- Supplementary Material

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Example of LTL : dictionary learning

Example : dictionary learning. The $X_{t,i} \in \mathbb{R}^K$, but all the relevant information is in $DX_{t,i} \in \mathbb{R}^k$, $k \ll K$. The matrix D is unknown.

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- $(X_{t,1}, Y_{t,1}), \dots, (X_{t,n}, Y_{t,n})$ i.i.d from P_{β_t} :

$$Y = \beta_t^T DX + \varepsilon,$$

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- $R_t(\beta, \Delta) = \mathbb{E}_{(X,Y)\sim P_{\beta_t}}[\ell(Y, \beta^T \Delta X)]$,

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- quantitative criterion to minimize w.r.t M

$$\mathcal{R}_{\mathrm{LTL}}(\Delta) = \mathbb{E}_{\beta\sim\mathcal{P}} \left\{ \mathbb{E}_{(X,Y)\sim P_\beta} \left[\ell(Y, \beta^T \Delta X) \right] \right\}.$$

Example of LTL : dictionary learning

Maurer, Pontil and Romera-Paredes propose :

$$\hat{D} = \arg\min_{\Delta} \sum_{t=1}^{M} \arg\min_{\|\beta_t\|_1 \leq \alpha} \sum_{i=1}^{n} \ell(Y_{t,i}, \beta_t^T \Delta X_{t,i})$$

Theorem (Maurer *et al*)

Under suitable assumptions, with probability at least $1-\delta$,

$$\mathcal{R}_{\mathrm{LTL}}(\hat{D}) \leq \inf_{\Delta} \mathcal{R}_{\mathrm{LTL}}(\Delta) + \mathcal{C}\left[\alpha k \sqrt{\frac{1}{M}} + \sqrt{\frac{\log\left(\frac{1}{\delta}\right)}{M}} + \alpha\sqrt{\frac{1}{n}}\right].$$

Note that $\mathcal{C}$ can depend on (k, K) or not, depending on assumptions on the distribution of X under P_β...

Going online : lifelong learning

Lifelong learning (LL)

Online version of learning-to-learn ?

Recent work with The Tien Mai and Massimiliano Pontil.
Objectives :

Going online : lifelong learning

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Objectives :

- consider that tasks can be revealed sequentially. Use the tools of online learning theory : avoid probabilistic assumptions.

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Online version of learning-to-learn ?

Recent work with The Tien Mai and Massimiliano Pontil.
Objectives :

- consider that tasks can be revealed sequentially. Use the tools of online learning theory : avoid probabilistic assumptions.
- if possible, define a general strategy that does not depend on the learning algorithm used within each task.

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Massimiliano Pontil
(UCL, IIT)

The Tien Mai
(U. of Oslo)

PMLR Proceedings of Machine Learning Research

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[edit]

Pierre Alquier, The Tien Mai, Massimiliano Pontil ; Proceedings of the 20th International Conference on Artificial Intelligence and Statistics, PMLR 54:261-269, 2017.

Abstract

We consider the problem of transfer learning in an online setting. Different tasks are presented sequentially and processed by a within-task algorithm. We propose a lifelong learning strategy which refines the underlying data representation used by the within-task algorithm, thereby transferring information from one task to the next. We show that when the within-task algorithm comes with some regret bound, our strategy inherits this good property. Our bounds are in expectation for a general loss function, and uniform for a convex loss. We discuss applications to dictionary learning and finite set of

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Propose initial g.

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2. udpate g.

Within-task algorithm

For $t = 1, 2, \ldots,$

1. Solve a usual online task, input $z_{t,i} = g(x_{t,i})$, output $y_{t,i}$.
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1. Solve a usual online task, input $z_{t,i} = g(x_{t,i})$, output $y_{t,i}$.
2. udpate g.

We can do it using any online algorithm. Will be refered to as "within-task algorithm".

For many algorithms, bounds are known on the (normalized)-regret :

$$\mathcal{R}_t(g) = \underbrace{\frac{1}{n_t}\sum_{i=1}^{n_t} \ell(y_{t,i}, \hat{y}_{t,i})}_{=\frac{1}{n_t}\sum_{i=1}^{n_t}\hat{\ell}_{t,i}=\hat{L}_t(g)} - \frac{1}{n_t}\inf_{h\in\mathcal{H}}\sum_{i=1}^{n_t} \ell(y_{t,i}, h(z_{t,i})).$$

Examples of within-task algorithms

Online gradient for convex ℓ

Initialize $h = 0$.
Update $h \leftarrow h - \eta \nabla_{f=h} \ell(y_{t,i}, f(z_{t,i}))$.

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Many variants and improvements (projected gradient, online Newton-step, ...).
$\mathcal{R}_t(g)$ in $1/\sqrt{n_t}$ or $1/n_t$ depending on assumptions on ℓ.

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EWA (Exponentially Weighted Aggregation)

Prior $\rho_1 = \pi$, initialize $h \sim \rho_1$.
Update $\rho_{i+1}(\mathrm{d}f) \propto \exp[-\eta \ell(y_{t,i}, f(z_{t,i}))] \rho_i(\mathrm{d}f)$, $h \sim \rho_{i+1}$.

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$\mathbb{E}[\mathcal{R}_t(g)]$ in $1/\sqrt{n_t}$ under boundedness assumption.
Integrated variant : $\mathcal{R}_t(g)$ in $1/n_t$ if ℓ is exp-concave.

EWA for lifelong learning

EWA-LL

Prior $\pi = \rho_1$ on $\mathcal{G}$. Draw $g \sim \pi$.
For $t = 1, 2, \dots$

1. run the within-task algorithm on task t. Suffer $\hat{L}_t(g)$.
2. update $\rho_{t+1}(\mathrm{d}f) \propto \exp[-\eta \hat{L}_t(f)]\rho_t(\mathrm{d}f)$.
3. draw $g \sim \rho_{t+1}$.

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Next : we provide two examples that are corollaries of a general result (stated later).

Example 1 : dictionary learning

$$\begin{array}{ccccc} \mathcal{X} = \mathbb{R}^K & \rightarrow & \mathcal{Z} = \mathbb{R}^k & \rightarrow & \mathcal{Y} = \mathbb{R} \\ x & \mapsto & Dx & \mapsto & \langle h, Dx \rangle = h^T Dx. \end{array}$$

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Theorem (Corollary 4.4) - ℓ is bounded by B & L-Lipschitz

$$\mathbb{E}\left[\frac{1}{T}\sum_{t=1}^{T}\frac{1}{n_t}\sum_{i=1}^{n_t}\hat{\ell}_{t,i}\right] \leq \inf_{D}\frac{1}{T}\sum_{t=1}^{T}\inf_{\|h_t\|\leq C}\frac{1}{n_t}\sum_{i=1}^{n_t}\ell(y_{t,i}, h_t^T D x_{t,i}) + \frac{C}{4}\sqrt{\frac{Kk}{T}}(\log(T)+7) + \frac{BL}{\sqrt{T}} + \frac{1}{T}\sum_{t=1}^{T}\frac{BL\sqrt{2k}}{\sqrt{n_t}}.$$

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$$\mathbb{E}\left[\frac{1}{T}\sum_{t=1}^{T}\frac{1}{n_t}\sum_{i=1}^{n_t}\hat{\ell}_{t,i}\right] \leq \inf_D \frac{1}{T}\sum_{t=1}^{T}\inf_{\|h_t\|\leq C}\frac{1}{n_t}\sum_{i=1}^{n_t}\ell(y_{t,i}, h_t^T D x_{t,i}) + \frac{C}{4}\sqrt{\frac{Kk}{T}}(\log(T)+7) + \frac{BL}{\sqrt{T}} + \frac{BL\sqrt{2k}}{\sqrt{\bar{n}}}.$$

Example 1 (dictionary learning) : simulations

- simulations $\mathcal{X} = \mathbb{R}^5 \to \mathcal{Z} = \mathbb{R}^2 \to \mathcal{Y} = \mathbb{R}$ with ℓ the quadratic loss, $T = 150$, each $n_t = 100$.

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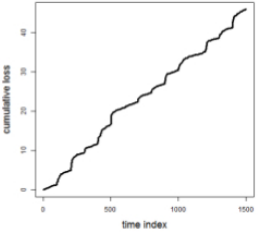

Figure 1: The cumulative loss of the oracle for the first 15 tasks.

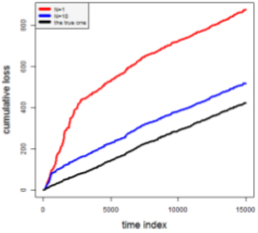

Figure 2: Cumulative loss of EWA-LL ($N = 1$ in red and $N = 10$ in blue) and cumulative loss of the oracle.

Example 2 : finite set of predictors

$$x \overset{g \in \mathcal{G}}{\mapsto} g(x) \overset{h \in \mathcal{H}}{\mapsto} h(g(x)).$$

$$\mathrm{card}(\mathcal{G}) = G < +\infty, \ \mathrm{card}(\mathcal{H}) = H < +\infty$$

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- within-task algorithm : EWA, uniform prior.
- EWA-LL, uniform prior.

Theorem (Corollary 4.2) - ℓ bounded by C & α-exp-concave

$$\mathbb{E}\left[\frac{1}{T}\sum_{t=1}^{T}\frac{1}{m}\sum_{i=1}^{m}\hat{\ell}_{t,i}\right] \leq \inf_{g\in\mathcal{G}}\frac{1}{T}\sum_{t=1}^{T}\inf_{h_t\in\mathcal{H}}\frac{1}{m}\sum_{i=1}^{m}\ell(y_{t,i}, h_t\circ g(x_{t,i})) + C\sqrt{\frac{\log G}{2T}} + \frac{\alpha \log H}{\bar{n}}.$$

Example 2 : improvement on existing results

The "online-to-batch" trick allows to deduce from our online method a statistical estimator with a controled LTL risk in

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In this case, a previous bound by Pentina and Lampert was in

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A PAC-Bayesian Bound for Lifelong Learning

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General regret bound

Theorem (Theorem 3.1) - ℓ bounded by C

If for any $g \in \mathcal{G}$, the within-task algorithm has a regret bound $\mathcal{R}_t(g) \leq \beta(g, n_t)$, then

$$\mathbb{E}\left[\frac{1}{T}\sum_{t=1}^{T}\frac{1}{n_t}\sum_{i=1}^{n_t}\hat{\ell}_{t,i}\right] \leq \inf_{\rho}\left\{\int\left[\frac{1}{T}\sum_{t=1}^{T}\inf_{h_t\in\mathcal{H}}\frac{1}{n_t}\sum_{i=1}^{n_t}\ell(y_{t,i}, h_t\circ g(x_{t,i})) + \frac{1}{T}\sum_{t=1}^{T}\beta(g, n_t)\right]\rho(\mathrm{d}g) + \frac{\eta C^2}{8} + \frac{\mathcal{K}(\rho,\pi)}{\eta T}\right\}.$$

1. Transfer learning, multitask learning, lifelong learning...
2. A strategy for lifelong learning, with regret analysis
3. Open questions

Efficient algorithms ?

Our online analysis allows to avoid explicit probabilistic assumptions on the data, and allows a free choice of the within-task algorithm.

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However, EWA-LL is not “truly online” as its computation requires to store all the data seen so far.

Moreover, its computation is not scalable.

Efficient Lifelong Learning Algorithm : ELLA

ELLA: An Efficient Lifelong Learning Algorithm

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1. Introduction

Versatile learning systems must be capable of efficiently and continually acquiring knowledge over a series of prediction tasks. In such a lifelong learning setting, the agent receives tasks sequentially. At any time, the agent may be asked to solve a problem from any previous task, and so must maximize its performance across all learned tasks at each step. When the solutions to these tasks are related through some underlying structure, the agent may share knowledge between tasks to improve learning performance, as explored in both transfer and multi-task learning.

Despite this commonality, current algorithms for transfer and multi-task learning are insufficient for lifelong learning. Transfer learning focuses on efficiently modeling a new target task by leveraging solutions to previously learned source tasks, without considering potential improvements to the source task models. In contrast, multi-task learning (MTL) focuses on maximizing performance across *all* tasks through shared knowledge, at potentially high computational cost. Lifelong learning includes elements of both paradigms, focusing on efficiently learning each consecutive task by building upon previous knowledge while optimizing performance across all tasks. In particular, lifelong learning incorporates the notion of *reverse transfer*, in which learning subsequent tasks can improve the performance of previously learned task models. Lifelong learning could also be considered as online MTL.

In this paper, we develop an Efficient Lifelong Learning Algorithm (ELLA) that incorporates aspects of both transfer and multi-task learning. ELLA learns and maintains a library of latent model components as a shared basis for all task models, supporting soft task grouping and overlap (Kumar & Daumé III, 2012). As each new task arrives, ELLA transfers knowledge through the shared basis to learn the new model, and refines the basis with knowledge from the new task. By refining the basis over time, newly acquired knowledge is integrated into existing basis vectors, thereby improving previously learned task models. This process is computationally efficient, and we provide robust theoretical guarantees on ELLA's performance and convergence. We evaluate ELLA on three challenging multi-task data sets: land mine detection, facial expression recognition, and student exam score prediction. Our results show that ELLA achieves nearly identical performance to batch MTL with three orders of magnitude (over 1,000x) speedup in learning time. We also compare ELLA to a current method for online MTL (Saha et al., 2011), and find that ELLA has both lower computational cost and higher performance.

2. Related Work

Early work on lifelong learning focused on sharing distance metrics using task clustering (Thrun & O'Sullivan, 1996), and transferring invariances in neu-

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More progress on dictionary learning

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arXiv:1803.08089v1 [stat.ML] 21 Mar 2018

Incremental Learning-to-Learn with Statistical Guarantees

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March 23, 2018

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1 INTRODUCTION

Learning-to-learn (LTL) or meta learning aims at finding an algorithm that is best suited to address a class of learning problems (tasks). These tasks are sampled from an unknown meta distribution and are only partially observed via a finite collection of training examples, see [6, 19, 36] and references therein. This problem plays a large role in artificial intelligence in that it can improve the efficiency of learning from human supervision. In particular, substantial improvement over "learning in isolation" (also known as independent task learning) is to be expected when the sample size per task is small, a setting which naturally arises in many applications [10, 30, 32, 37].
LTL is particularly appealing when considered from an online or incremental perspective. In this setting, which is sometimes referred to as lifelong learning (see, e.g. [33]), the tasks are observed sequentially – via corresponding sets of training examples – from a common environment and we aim to improve the learning ability of the underlying algorithm on future yet-to-be-seen tasks from the same environment. Practical scenarios of lifelong learning are wide ranging, including computer vision [30], robotics [10], user modelling and many more.

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In learning-to-learn the goal is to infer a learning algorithm that works well on a class of tasks sampled from an unknown meta distribution. In contrast to previous work on batch learning-to-learn, we consider a scenario where tasks are presented sequentially and the algorithm needs to adapt incrementally to improve its performance on future tasks. Key to this setting is for the algorithm to rapidly incorporate new observations into the model as they arrive, without keeping them in memory. We focus on the case where the underlying algorithm is Ridge Regression parameterized by a positive semidefinite matrix. We propose to learn this matrix by applying a stochastic strategy to minimize the empirical error incurred by Ridge Regression on future tasks sampled from the meta distribution. We study the statistical properties of the proposed algorithm and prove non-asymptotic bounds on its excess transfer risk, that is, the generalization performance on new tasks from the same meta distribution. We compare our online learning-to-learn approach with a state of the art batch method, both theoretically and empirically.

1 INTRODUCTION

Learning-to-learn (LTL) or meta learning aims at finding an algorithm that is best suited to address a class of learning problems (tasks). These tasks are sampled from an unknown meta distribution and are only partially observed via a finite collection of training examples, see [6, 19, 36] and references therein. This problem plays a large role in artificial intelligence in that it can improve the efficiency of learning from human supervision. In particular, substantial improvement over "learning in isolation" (also known as independent task learning) is to be expected when the sample size per task is small, a setting which naturally arises in many applications [10, 30, 32, 37].

LTL is particularly appealing when considered from an online or incremental perspective. In this setting, which is sometimes referred to as lifelong learning (see, e.g. [33]), the tasks are observed sequentially – via corresponding sets of training examples – from a common environment and we aim to improve the learning ability of the underlying algorithm on future yet-to-be-seen tasks from the same environment. Practical scenarios of lifelong learning are wide ranging, including computer vision [30], robotics [10], user modelling and many more.

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- dictionary learning,
- fast update of β at each step, fast update of D at the end of each task, truly online,
- very good empirical performances,
- LTL bound in

$$\mathcal{O}\left(\sqrt{\frac{1}{T}}+\sqrt{\frac{1}{n}}\right).$$

Algorithms : open questions

Open question 1

An efficient algorithm with theoretical guarantees (if possible beyond dictionary learning).

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- theoretical analysis of ELLA ?
- can we justify to update D at each step ? this leads to the next big open problem...

Optimality of the bounds

- ELLA : updates D at each step. Doing so, after T tasks with n steps in each task, we would expect a bound in

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So, what are the optimal rates in LL & LTL ?

Insights from a toy model

- θ_1 fixed once and for all,
- task t : $\theta_{2,t}$ fixed for the task
- for $i = 1, \dots, n$, $y_{t,i} = (\theta_1 + \varepsilon_{1,i,t}, \theta_{2,t} + \varepsilon_{2,i,t})$ with $\varepsilon_{j,i,t} \sim \mathcal{N}(0,1)$.

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$\hat{\theta}_1 = \frac{1}{nT} \sum_{t=1}^{T} \sum_{i=1}^{n} (y_{t,i})_1$ can be computed in the online setting and one has

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Fits our setting with $x = \emptyset$, $g_{\theta_1}(x) = \theta_1$, $h_{\theta_2}(z) = (z, \theta_2)$.

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Still fits our setting and LTL !

Optimal rates : open questions

Open question 2

What are the optimal rates in lifelong learning and in LTL ?

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Optimal rates : open questions

Open question 2

What are the optimal rates in lifelong learning and in LTL ?

- requires to define properly class of predictors,
- the optimal rate will also depend on the setting. This leads to the next question...

Are our definitions even right ?

- Note that the terminology is not exen fixed : for example, Pentina and Lampert call lifelong learning what we call learning to learn (we don't claim we are right !).

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- We used :
    1. LTL : samples from all the tasks presented at once.
    2. LL : tasks presented sequentially, within each task, pairs presented sequentially.
    3. why not tasks presented sequentially, but within each task, samples presented all at once ? .

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- We used :
  1. LTL : "Batch-within-batch"
  2. LL : "Online-within-online"
  3. "Batch-within-online", see our paper and Denivi *et al*.

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Do we really need a paper for each possible variant ?...

Setting : open questions

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- depends on the applications.
- should also have a look on other existing approaches (econometrics of panel data $\leftrightarrow$ multitask learning).