Concentration and robustness of discrepancy-based ABC

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Advanced Intelligence Project

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The Maximum Likelihood Estimator (MLE)

Let $X_1, \dots, X_n$ be i.i.d in $\mathcal{X}$ from a probability distribution P_0.

Statistical inference :

- propose a model $(P_\theta, \theta \in \Theta)$, assume $P_0 = P_{\theta_0}$.
- compute $\hat{\theta}_n = \hat{\theta}_n(X_1, \dots, X_n)$.

Letting p_θ denote the density of P_θ, then

$$\hat{\theta}_n^{MLE} = \arg\max_{\theta \in \Theta} L_n(\theta), \text{ where } L_n(\theta) = \prod_{i=1}^n p_\theta(X_i).$$

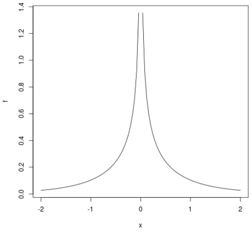
Example : $P_{(m,\sigma)} = \mathcal{N}(m, \sigma^2)$ then

$$\hat{m} = \frac{1}{n} \sum_{i=1}^n X_i \text{ and } \hat{\sigma}^2 = \frac{1}{n} \sum_{i=1}^n (X_i - \hat{m})^2.$$

MLE not unique / not consistent

Example :

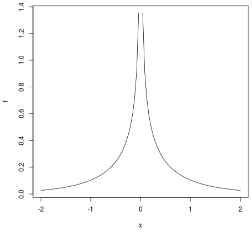
$$p_\theta(x) = \frac{\exp(-|x-\theta|)}{2\sqrt{\pi|x-\theta|}},$$



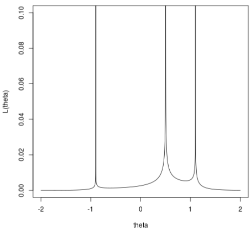
MLE not unique / not consistent

Example :

$$p_\theta(x) = \frac{\exp(-|x-\theta|)}{2\sqrt{\pi|x-\theta|}},$$



$$L_n(\theta) = \frac{\exp\left(-\sum_{i=1}^n |X_i-\theta|\right)}{(2\sqrt{\pi})^n \prod_{i=1}^n \sqrt{|X_i-\theta|}}.$$



MLE fails in the presence of outliers

What is an outlier?

Huber proposed the contamination model : with probability ε, X_i is not drawn from P_{θ_0} but from Q that can be anything :

$$P_0 = (1-\varepsilon)P_{\theta_0} + \varepsilon Q.$$

Example : $P_\theta = \mathcal{U}nif[0,\theta]$, then

$$L_n(\theta) = \frac{1}{\theta^n}\prod_{i=1}^{n} \mathbb{1}_{\{0 \leq X_i \leq \theta\}} \Rightarrow \hat{\theta} = \max_{1\leq i\leq n} X_i.$$

In the case of the following contamination, the MLE is extremely far from the truth :

$$P_0 = (1-\varepsilon).\mathcal{U}nif[0,1] + \varepsilon.\mathcal{N}(10^{10},1)...$$

Minimum Distance Estimation

$$\text{Empirical distribution}: \hat{P}_n := \frac{1}{n}\sum_{i=1}^{n} \delta_{X_i}.$$

Minimum Distance Estimation (MDE)

Let $d(\cdot,\cdot)$ be a metric on probability distributions.

$$\hat{\theta}_d := \arg\min_{\theta\in\Theta} d\left(P_\theta, \hat{P}_n\right).$$

Wolfowitz, J. (1957). The minimum distance method. *The Annals of Mathematical Statistics*.

Idea : MDE with an adequate d leads to robust estimation.

Bickel, P. J. (1976). Another look at robustness : a review of reviews and some new developments. *Scandinavian Journal of Statistics*. Discussion by Sture Holm.

Parr, W. C. & Schucany, W. R. (1980). Minimum distance and robust estimation. *JASA*.

Yatracos, Y. G. (1985). Rates of convergence of minimum distance estimators and Kolmogorov's entropy. *Annals of Statistics*.

Integral Probability Semimetrics

Integral Probability Semimetrics (IPS)

Let $\mathcal{F}$ be a set of real-valued, measurable functions and put

$$d_{\mathcal{F}}(P,Q) = \sup_{f\in\mathcal{F}} \left| \mathbb{E}_{X\sim P}[f(X)] - \mathbb{E}_{X\sim Q}[f(X)] \right|.$$

Müller, A. (1997). Integral probability metrics and their generating classes of functions. *Applied Probability*.

- assumptions required in order to ensure that $d_{\mathcal{F}}(P,Q) = 0 \Rightarrow P = Q$ (that is, $d_{\mathcal{F}}$ is a metric).
- assumptions required in order to ensure that $d_{\mathcal{F}} < +\infty$.

Non-asymptotic bound for MDE

Theorem 1

- $X_1, \dots, X_n$ i.i.d from P_0,
- for any $f \in \mathcal{F}$, $\sup_{x \in \mathcal{X}} |f(x)| \leq 1$.

Then

$$\mathbb{E}\left[d_{\mathcal{F}}(P_{\hat{\theta}_{d_{\mathcal{F}}}}, P_0)\right] \leq \inf_{\theta \in \Theta} d_{\mathcal{F}}(P_\theta, P_0) + 4.\mathrm{Rad}_n(\mathcal{F}).$$

Rademacher complexity

$$\mathrm{Rad}_n(\mathcal{F}) := \sup_P \mathbb{E}_{Y_1,\dots,Y_n \sim P} \mathbb{E}_{\epsilon_1,\dots,\epsilon_n}\left[\sup_{f \in \mathcal{F}} \frac{1}{n} \sum_{i=1}^n \epsilon_i f(Y_i)\right].$$

where $\epsilon_1, \dots, \epsilon_n$ are i.i.d Rademacher variables :

$$\mathbb{P}(\epsilon_1 = 1) = \mathbb{P}(\epsilon_1 = -1) = 1/2.$$

Example 1 : set of indicators

$$\mathbb{1}_A(x) = \begin{cases} 1 \text{ if } x \in A, \\ 0 \text{ if } x \notin A. \end{cases}$$

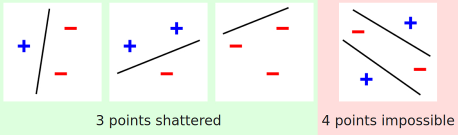


Image from Wikipedia.

Reminder - Vapnik-Chervonenkis dimension

Assume that $\mathcal{F} = \{\mathbb{1}_A, A \in \mathcal{A}\}$ for some $\mathcal{A} \subseteq \mathcal{P}(\mathcal{X})$,

- $S_{\mathcal{F}}(x_1, \dots, x_n) := \{(f(x_1), \dots, f(x_n)), f \in \mathcal{F}\}$,
- $\mathrm{VC}(\mathcal{F}) := \max\{n : \exists x_1, \dots, x_n, |S_{\mathcal{F}}(x_1, \dots, x_n)| = 2^n\}$.

Theorem (Bartlett and Mendelson)

$$\mathrm{Rad}_n(\mathcal{F}) \leq \sqrt{\frac{2.\mathrm{VC}(\mathcal{F}) \log(n+1)}{n}}.$$

Bartlett, P. L. & Mendelson, S. (2002). Rademacher and Gaussian complexities : Risk bounds and structural results. *JMLR*.

Example 1 : KS and TV distances

Two classical examples :

- $\mathcal{A} = \{$all measurable sets in $\mathcal{X}\}$, then $d_{\mathcal{F}}(\cdot,\cdot)$ is the total variation distance $\mathrm{TV}(\cdot,\cdot)$.
  - $\mathrm{VC}(\mathcal{F}) = +\infty$ when $|\mathcal{X}| = +\infty$,
  - in general, $\mathrm{Rad}_n(\mathcal{F}) \nrightarrow 0$.
- $\mathcal{X} = \mathbb{R}$, $\mathcal{A} = \{(-\infty, x], x \in \mathbb{R}\}$, then $d_{\mathcal{F}}(\cdot,\cdot)$ is the Kolmogorov-Smirnov distance $\mathrm{KS}(\cdot,\cdot)$.
  - KS distance was actually proposed by S. Holm for robust estimation,
  - $\mathrm{VC}(\mathcal{F}) = 1$.

$$\mathbb{E}\left[\mathrm{KS}(P_{\hat{\theta}_{\mathrm{KS}}}, P_0)\right] \leq \inf_{\theta\in\Theta} \mathrm{KS}(P_\theta, P_0) + 4.\sqrt{\frac{2\log(n+1)}{n}}.$$

Example 2 : Maximum Mean Discrepancy (MMD)

- Let $(\mathcal{H}, \langle \cdot, \cdot \rangle_{\mathcal{H}})$ be a RKHS with kernel
$$k(x, y) = \langle \phi(x), \phi(y) \rangle_{\mathcal{H}} .$$
- If $\|\phi(x)\|_{\mathcal{H}} = k(x, x) \leq 1$ then $\mathbb{E}_{X \sim P}[\phi(X)]$ is well-defined .
- The map $P \mapsto \mathbb{E}_{X \sim P}[\phi(X)]$ is one-to-one if k is *characteristic*.
- For example, $k(x, y) = \exp(-\|x - y\|^2/\gamma^2)$ works.

Definition - MMD

$$\begin{aligned}\mathrm{MMD}_k(P, Q) &= \sup_{\substack{f \in \mathcal{H} \\ \|f\|_{\mathcal{H}} \leq 1}} \left| \mathbb{E}_{X \sim P}[f(X)] - \mathbb{E}_{X \sim Q}[f(X)] \right| \\ &= \left\| \mathbb{E}_{X \sim P}[\phi(X)] - \mathbb{E}_{X \sim Q}[\phi(X)] \right\|_{\mathcal{H}} .\end{aligned}$$

Example 2 : MMD

$$\mathcal{F} = \{f \in \mathcal{H} : \|f\|_{\mathcal{H}} \leq 1\} \Rightarrow \mathrm{Rad}_n(\mathcal{F}) \leq \sqrt{\frac{\sup_x k(x,x)}{n}}.$$

Theorem 2

For k bounded by 1 and characteristic,

$$\mathbb{E}\left[\mathrm{MMD}_k(P_{\hat{\theta}_{\mathrm{MMD}_k}}, P_0)\right] \leq \inf_{\theta\in\Theta} \mathrm{MMD}_k(P_\theta, P_0) + \frac{2}{\sqrt{n}}.$$

Joint work with Badr-Eddine Chérief-Abdellatif (Oxford).

Chérief-Abdellatif, B.-E. and Alquier, P. Finite Sample Properties of Parametric MMD Estimation : Robustness to Misspecification and Dependence. **Bernoulli**, 2022.

Example 2 : MMD

We actually have

$$\mathrm{MMD}_k^2(P_\theta, \hat{P}_n) = \mathbb{E}_{X,X'\sim P_\theta}[k(X,X')] - \frac{2}{n}\sum_{i=1}^{n} \mathbb{E}_{X\sim P_\theta}[k(X_i,X)] + \frac{1}{n^2}\sum_{1\leq i,j\leq n} k(X_i,X_j)$$

and so

$$\nabla_\theta \mathrm{MMD}_k^2(P_\theta, \hat{P}_n) = 2\mathbb{E}_{X,X'\sim P_\theta}\left\{\left[k(X,X') - \frac{1}{n}\sum_{i=1}^{n} k(X_i,X)\right]\nabla_\theta[\log p_\theta(X)]\right\}$$

that can be approximated by sampling from P_θ.

Example 2 : MMD

Dziugaite, G. K., Roy, D. M., & Ghahramani, Z. (2015). Training generative neural networks via maximum mean discrepancy optimization. *UAI 2015*.

define the estimator and used it to train GANs.

Briol, F. X., Barp, A., Duncan, A. B., & Girolami, M. (2019). Statistical Inference for Generative Models with Maximum Mean Discrepancy. *Preprint arXiv :1906.05944*.

$$\text{assumptions} \;\Rightarrow\; \sqrt{n}(\hat{\theta}_{\mathrm{MMD}_k} - \theta_0) \rightsquigarrow \mathcal{N}(0, V_0(k)).$$

Example 3 : Wasserstein

Another classical metric belongs to the IPS family :

$$\mathrm{W}_\delta(P,Q) = \sup_{\substack{f:\mathcal{X}\to\mathbb{R} \\ \mathrm{Lip}(f)\leq 1}} \left| \mathbb{E}_{X\sim P}[f(X)] - \mathbb{E}_{X\sim Q}[f(X)] \right|$$

where $\mathrm{Lip}(f) := \sup_{x\neq y} |f(x)-f(y)|/\delta(x,y)$.

- In general, $\mathrm{Rad}_n(\mathcal{F}) \not\to 0$, so will not converge in full generality as with MMD and KS.
- However, nice results can be proven under additional assumptions :

Bernton, E., Jacob, P. E., Gerber, M. & Robert, C. P. (2019). On parameter estimation with the Wasserstein distance. *Information and Inference : A Journal of the IMA*.

MDE and robustness

Reminder

$$\mathbb{E}\left[d_{\mathcal{F}}(P_{\hat{\theta}_{d_{\mathcal{F}}}}, P_0)\right] \leq \inf_{\theta\in\Theta} d_{\mathcal{F}}(P_\theta, P_0) + 4.\mathrm{Rad}_n(\mathcal{F}).$$

Huber's contamination model : $P_0 = (1-\varepsilon)P_{\theta_0} + \varepsilon Q$.

$$\begin{aligned}
d_{\mathcal{F}}(P_{\theta_0}, P_0) &= \sup_{f\in\mathcal{F}} \left|\mathbb{E}_{X\sim P_{\theta_0}} f(X) - (1-\varepsilon)\mathbb{E}_{X\sim P_{\theta_0}} f(X) - \varepsilon \mathbb{E}_{X\sim Q} f(X)\right| \\
&= \sup_{f\in\mathcal{F}} \left|\varepsilon \mathbb{E}_{X\sim P_{\theta_0}} f(X) - \varepsilon \mathbb{E}_{X\sim Q} f(X)\right| \\
&= \varepsilon . d_{\mathcal{F}}(P_{\theta_0}, Q) \leq 2\varepsilon \quad \text{if for any } f\in\mathcal{F}, \sup_x |f(x)| \leq 1
\end{aligned}$$

Corollary - in Huber's contamination model

$$\mathbb{E}\left[d_{\mathcal{F}}(P_{\hat{\theta}_{d_{\mathcal{F}}}}, P_{\theta_0})\right] \leq 4\varepsilon + 4.\mathrm{Rad}_n(\mathcal{F}).$$

MDE and robustness : toy experiment

Model : $\mathcal{N}(\theta, 1)$, $X_1, \dots, X_n$ i.i.d $\mathcal{N}(\theta_0, 1)$, $n = 100$ and we repeat the exp. 200 times. Kernel $k(x, y) = \exp(-|x - y|)$.

	$\hat{\theta}_{MLE}$	$\hat{\theta}_{\mathrm{MMD}_k}$	$\hat{\theta}_{\mathrm{KS}}$
mean abs. error	0.081	0.094	0.088

Now, $\varepsilon = 2\%$ of the observations drawn from a Cauchy.

mean abs. error	0.276	0.095	0.088

Now, $\varepsilon = 1\%$ are replaced by $1,000$.

mean abs. error	10.008	0.088	0.082

Contents

Generalized posteriors

Posterior

$$\pi(\theta|X_1,\ldots,X_n) \propto L_n(\theta)\pi(\theta).$$

Generalized posterior

$$\hat{\pi}_{\beta,R_n}(\theta) \propto \exp(-\beta.R_n(\theta))\pi(\theta).$$

- old idea in ML (PAC-Bayes, forecasting with expert advice...) and in statistics (Gibbs posteriors...)
- popularized / extended and studied by :

Bissiri, P. G., Holmes, C. C. & Walker, S. G. (2016). A general framework for updating belief distributions. *JRSS-B*.

Knoblauch, J., Jewson, J. & Damoulas, T. (2022). An Optimization-centric View on Bayes' Rule : Reviewing and Generalizing Variational Inference. *JMLR* (to appear).

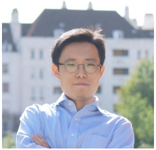
Generalizing the posterior with IPS

Generalized posterior with IPS

$$\hat{\pi}_{\beta,R_n}(\theta) \propto \exp(-\beta.d_{\mathcal{F}}(P_\theta, \hat{P}_n))\pi(\theta).$$

- in the MMD case : non-asymptotic result in

Chérief-Abdellatif, B.-E. and Alquier, P. (2020). MMD-Bayes : Robust Bayesian Estimation via Maximum Mean Discrepancy. *Proceedings of AABI.*



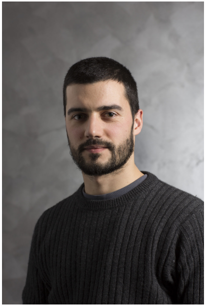
- asymptotic results to come very soon in a joint paper with Takuo Matsubara (Newcastle) and Jeremias Knoblauch (UCL).
- both papers discuss computation via variational approximations or MCMC.

Contents

ABC with IPS

What follows is based on a joint work with :

Sirio Legramanti
(University of Bergamo)



Daniele Durante
(Bocconi University, Milan)

Reminder on ABC

Approximate Bayesian Computation (ABC)

input : sample $X_1^n = (X_1, \dots, X_n)$, model $(P_\theta, \theta \in \Theta)$, prior π, statistic S, distance δ and threshold ϵ.

(i) sample $\theta \sim \pi$,

(ii) sample $Y_1^n = (Y_1, \dots, Y_n)$ i.i.d. from P_θ :
  - if $\delta(S(X_1^n), S(Y_1^n)) \leq \epsilon$ return θ,
  - else goto (i).

- how close is the distribution of the output to the posterior $\pi(\theta|X_1, \dots, X_n)$?
- reverse point of view : what are the properties of the "generalized posterior" we sample from ?

ABC with IPS

Here, we study the situation :

- $S(x_1, \ldots, x_n) = \frac{1}{n}\sum_{i=1}^n \delta_{x_i}$ the empirical distribution,
- $\delta(P, Q) = d_{\mathcal{F}}(P, Q)$.

IPS–ABC

input : sample $X_1^n = (X_1, \ldots, X_n)$, model $(P_\theta, \theta \in \Theta)$, prior π, set of functions $\mathcal{F}$ and threshold ϵ. Put $\hat{P}_n = \frac{1}{n}\sum_{i=1}^n \delta_{X_i}$.

(i) sample $\theta \sim \pi$,

(ii) sample $Y_1^n = (Y_1, \ldots, Y_n)$ i.i.d. from P_θ and put $\hat{P}_n^Y = \frac{1}{n}\sum_{i=1}^n \delta_{Y_i}$,

- if $d_{\mathcal{F}}(\hat{P}_n, \hat{P}_n^Y) \leq \epsilon$ return θ,
- else goto (i).

Notation : the output $\vartheta \sim \hat{\pi}_{n,\epsilon}^{\mathcal{F}}(\cdot)$.

Properties of $\hat{\pi}^{\mathcal{F}}_{n,\epsilon}(\cdot)$

3 questions :

1. $\hat{\pi}^{\mathcal{F}}_{n,\epsilon}(\theta) \xrightarrow[\epsilon \searrow ?]{?} \pi(\theta|X_1^n)$.
2. $\hat{\pi}^{\mathcal{F}}_{n,\epsilon}(\theta) \xrightarrow[n\to\infty]{?} ?$
3. $\hat{\pi}^{\mathcal{F}}_{n,\epsilon_n}(\cdot) \xrightarrow[n\to\infty]{?} \delta_{\theta_0}$ if $P_0 = P_{\theta_0}$.

Contraction of the ABC posterior

$$\epsilon_* := \inf_{\theta \in \Theta} d_{\mathcal{F}}(P_\theta, P_0).$$

Theorem 3

Assume :

- for all $\epsilon > 0$, $\pi(\{\theta : d_{\mathcal{F}}(P_\theta, P_0) \leq \epsilon_* + \epsilon\}) \geq c\epsilon^d$.
- $\forall f \in \mathcal{F}$, $\sup_{x \in \mathcal{X}} |f(x)| \leq 1$.
- $\mathrm{Rad}_n(\mathcal{F}) \xrightarrow[n \to \infty]{} 0$.

Let ϵ_n be any sequence such that $\epsilon_n / \mathrm{Rad}_n(\mathcal{F}) \to \infty$ and $n\epsilon_n^2 \to \infty$. Then, with probability $\to 1$ on the sample, for any $M_n \to \infty$,

$$\hat{\pi}^{\mathcal{F}}_{n,\epsilon_*+\epsilon_n}\left(d_{\mathcal{F}}(P_\theta, P_0) \leq \epsilon_* + \frac{4\epsilon_n}{3} + \mathrm{Rad}_n(\mathcal{F}) + \sqrt{\frac{\log \frac{M_n}{\epsilon_n^d}}{n}} \right) \geq 1 - \frac{2.3^d}{cM_n}.$$

Example : MMD-ABC with bounded kernel

As an example, consider MMD_k when $k(x,x) \leq 1$, as

Park, M., Jitkrittum, W. & Sejdinovic, D. (2016). K2-ABC : Approximate Bayesian Computation with kernel embeddings. *AISTATS*.

Corollary

Assume :

- for all $\epsilon > 0$, $\pi(\{\theta : d_{\mathcal{F}}(P_\theta, P_0) \leq \epsilon_* + \epsilon\}) \geq c\epsilon^d$.

Let $1/\sqrt{n} \ll \epsilon_n \ll 1$. Then, with probability $\to 1$ on the sample, for any $M_n \to \infty$,

$$\hat{\pi}^{\mathcal{F}}_{n,\epsilon_*+\epsilon_n}\left(\mathrm{MMD}_k(P_\theta, P_0) \leq \epsilon_* + \frac{4\epsilon_n}{3} + \frac{1+\sqrt{\log\frac{M_n}{\epsilon_n^d}}}{\sqrt{n}}\right) \geq 1 - \frac{2.3^d}{cM_n}.$$

A result without Rademacher complexity

Theorem 4

Assume :

- for all $\epsilon > 0$, $\pi(\{\theta : d_{\mathcal{F}}(P_\theta, P_0) \leq \epsilon_* + \epsilon\}) \geq c\epsilon^d$,
- $d_{\mathcal{F}}(\hat{P}_n, P_0) \xrightarrow{P_0 \text{ a.s.}} 0$,
- $\mathbb{P}_{Y_1^n \sim P_\theta}(d_{\mathcal{F}}(\hat{P}_n^Y, P_\theta) > \epsilon) \leq c(\theta) f_n(\epsilon)$ where $f_n(\epsilon) \xrightarrow[n\to\infty]{} 0$.

Let $\epsilon_n \to 0$ and $f_n(\epsilon_n) \to 0$. Then, with probability $\to 1$ on the sample, for some $C > 0$ and any $M_n \to \infty$,

$$\hat{\pi}^{\mathcal{F}}_{n,\epsilon_*+\epsilon_n}\left(d_{\mathcal{F}}(P_\theta, P_0) \leq \epsilon_* + \frac{4\epsilon_n}{3} + f_n^{-1}\left(\frac{\epsilon_n^d}{M_n}\right)\right) \geq 1 - \frac{C}{M_n}.$$

Example : MMD-ABC with unbounded kernel

Corollary

Assume :

- for all $\epsilon > 0$, $\pi(\{\theta : d_{\mathcal{F}}(P_\theta, P_0) \leq \epsilon_* + \epsilon\}) \geq c\epsilon^d$,
- $\mathbb{E}_{Z\sim Q}[k(Z,Z)] < +\infty$ for $Q = P_0$ and any $Q \in \{P_\theta, \theta \in \Theta\}$.

Let $1/n^{2d} \ll \epsilon_n \ll 1$. Then, with probability $\to 1$ on the sample, there is a $C > 0$ such that for any $M_n \to \infty$,

$$\hat{\pi}^{\mathcal{F}}_{n,\epsilon_*+\epsilon_n}\left(\mathrm{MMD}_k(P_\theta, P_0) \leq \epsilon_* + \frac{4\epsilon_n}{3} + \frac{1}{n}\sqrt{\frac{M_n}{\epsilon_n^d}}\right) \geq 1 - \frac{C}{M_n}.$$

Example : Wasserstein-ABC

Bernton, E., Jacob, P. E., Gerber, M. & Robert, C. P. (2019). Approximate Bayesian Computation with the Wasserstein distance. *JRSS-B*.

considered ABC with the Wasserstein distance, and proved Theorem 4 in this case (note that our Theorem 4 is a restatement of their result for a general IPS).

However, it not easy to prove non-trivial bounds :

$$\mathbb{P}_{Y_1^n \sim P_\theta}(d_{\mathcal{F}}(\hat{P}_n^Y, P_\theta) > \epsilon) \leq c(\theta) f_n(\epsilon),$$

and the examples they cite require $\mathcal{X}$ to be a **bounded space**.

Weed, J. & Bach, F. (2019). Sharp asymptotic and finite-sample rates of convergence of empirical measures in Wasserstein distance. *Bernoulli*.

$n \to \infty$, fixed ϵ

Jiang, B., Wu, T.-Y. & Wong, W. H. (2018). Approximate Bayesian computation with Kullback-Leibler divergence as data discrepancy. *AISTATS*.

provides a general result that can be directly used here :

Theorem (simplified version)

Assume :

- $\mathrm{Rad}_n(\mathcal{F}) \to 0$,

then for any measurable B,

$$\hat{\pi}^{\mathcal{F}}_{n,\epsilon}(\theta \in B) \xrightarrow[n\to\infty]{\text{a.s.}} \pi(\theta \in B | d_{\mathcal{F}}(P_\theta, P_0) \leq \epsilon_* + \epsilon).$$

$\epsilon \searrow 0$, fixed n

Bernton, E., Jacob, P. E., Gerber, M. & Robert, C. P. (2019). Approximate Bayesian Computation with the Wasserstein distance. *JRSS-B*.

provides a general result that can be directly used here :

Theorem (simplified version)

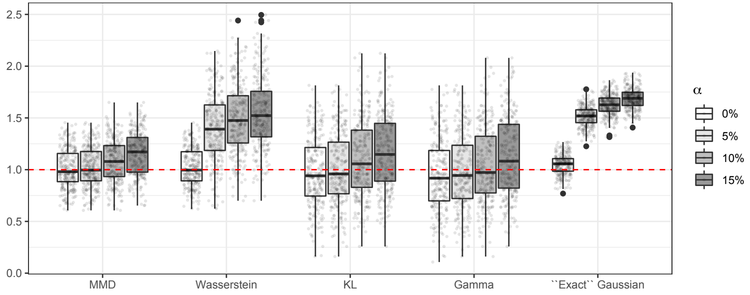
Assume :

- P_θ has continuous, bounded densities p_θ, and $P_0 = P_{\theta_0}$,
- $d_{\mathcal{F}}$ is continous, and is a metric,

then a.s. with respect to the sample, for any measurable B,

$$\hat{\pi}^{\mathcal{F}}_{n,\epsilon}(\theta \in B) \xrightarrow[\epsilon \searrow 0]{} \pi(\theta \in B | X_1^n).$$

Experimental results



Other current and future directions

- go beyond IPS, see (among others) f-divergences or energy statistics in

Frazier, D. T. (2020). *Robust and efficient Approximate Bayesian Computation : A minimum distance approach*. Preprint arXiv.

Nguyen, H. D., Arbel, J., Lü, H. and Forbes, F. (2020). Approximate Bayesian computation via the energy statistic. *IEEE Access*.

- solve practical issues : choice of ϵ_n, choice of k in MMD_k.
- semi-parametric models (with J.-D. Fermanian (ENSAE Paris), A. Derumigny (TU Delft) and M. Gerber (Bristol).



Alquier, P., Chérief-Abdellatif, B.-E., Derumigny, A. and Fermanian, J.-D. Estimation of copulas via Maximum Mean Discrepancy. JASA, to appear.

Alquier, P. and Gerber, M. (2020). *Universal Robust Regression via Maximum Mean Discrepancy*. Preprint arXiv.

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終わり

ありがとう ございます。